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A Model of Weak Interaction Based on the Three-Triplet Model of Hadrons. II

——High-Energy Neutrino Inclusive Reactions——

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High-energy neutrino inclusive reactions are discussed in the framework of the weak interaction model we proposed in a previous paper based on the three-triplet model. We get several scaling function sum rules which are different from those derived from the simple quark model in numerical values. These differences are due to the contributions from the unobserved final hadronic states which comprise super hadrons.

§ 1. Introduction

In a previous paper[1] (hereafter called Paper I), we have suggested an important role of the internal degrees of freedom involved in the three-triplet model[2] for the weak interaction and proposed a weak interaction model. In our model, the weak charged current is the so-called "*double* $V-A$ *type*" one[3] (its hadronic part is composed of three-triplet fields) and the weak interaction Hamiltonian ($\mathcal{H}_W$) is a current-current type. We have shown that our $\mathcal{H}_W$ has several characteristic properties; it leads to the $|\Delta I|=1/2$ rule in $|\Delta Y|=1$ nonleptonic decays of ordinary hadrons (assumed to belong to the $SU(3)''$-singlet) and involves no diagonal interaction terms in purely leptonic interactions. In this model, however, the number of leptonic fields is four (i.e., e, μ, ν_e and ν_μ), so the lepton-urbaryon (three-triplet) correspondence is somewhat obscure and some of the three-triplet fields (t_{ij}) do not take part in the semileptonic interaction. The charm-number changing semileptonic interaction is ambiguous in its form. Thus, it seems to be favourable to introduce nine kinds of leptons such as l_{ij} in § 6 of Paper I, also from the viewpoint of the lepton-urbaryon symmetry,[4] if we can explain the fact that the additional leptons have not yet been detected. When we introduce l_{ij} simply corresponding to t_{ij}, the internal degrees of freedom involved in the three-triplet model can be fully utilized and moreover essentially the same results as those of the originally proposed model[1] are also obtained ($|\Delta I|=1/2$ rule etc.). Therefore, we proceed with our considerations employing the model (a) given in § 6 of Paper I and a possible interpretation for the lack of evidence of our hypothetical leptons will be stated later in this paper.

It is the purpose of this paper to investigate what observable effects are derived in high-energy neutrino inclusive reactions[5] from our interaction. We carry out this purpose by calculating the light-cone (L-C) commutators[6] of the various weak hadronic currents [they have different forms depending on the leptonic currents they couple to (see § 2)], and by studying the behavior of structure functions in the Bjorken limit.[7] The scaling of the structure functions are obtained as a trivial result of the assumption of scale invariance on the L-C.[8] Moreover, we can get several sum rules which discriminate our model from the ordinary theory.[9] In particular, the Adler[10] and the Gross-Smith[11] type sum rules derived from our theory are really different from those of the quark model in numerical values. These discrepancies are due to the contributions from the final hadronic states which belong to **8** representation of the $SU(3)''$.

In § 2, we briefly summarize the model given in § 6 of Paper I and write down explicit forms of the semileptonic interaction. In § 3, the neutrino inclusive reactions with a nucleon target are considered. The L-C commutators for the $(\mathbf{1}+\mathbf{8}, \mathbf{1}+\mathbf{8})$ currents of $SU(3)' \times SU(3)''$ abstracted from the free three-triplet model are given, and sum rules for the scaling functions are derived. Remarks on our results and a possible interpretation of the situation that five additional leptons have not yet been detected are given in § 4.

§ 2. Weak charged current and interaction Hamiltonian

The weak charged current $\mathcal{J}_\rho^{(i)}(x)$ $(\Delta Q=1)$ introduced in Paper I, which is symmetric between its leptonic and hadronic parts, is written as follows:

$$\mathcal{J}_\rho^{(i)}(x) = j_\rho^{(i)}(x) + J_\rho^{(i)}(x), \tag{2·1}$$

where

$$\begin{aligned} J_\rho^{(i)} &\equiv g\bar{t}_{kl} O_\rho \left[\cos\theta_C \left(\frac{\lambda^1+i\lambda^2}{2}\right)_{km} + \sin\theta_C \left(\frac{\lambda^4+i\lambda^5}{2}\right)_{km}\right] (\Lambda^{(i)})_{lm} t_{mn}\,; \\ O_\rho &\equiv i\gamma_\rho(1+\gamma_5),\quad g\text{: const}, \\ \Lambda^{(i)} &= \Lambda^i \ \text{ for } i=1, 2, \cdots, 8;\ \Lambda^{(0)} = i\sqrt{2}\Lambda^0,^{*)} \end{aligned} \tag{2·2}$$

is the weak hadronic current and we take θ_C the Cabibbo angle, and

$$j_\rho^{(i)} \equiv \bar{l}_{kl} O_\rho \left[\cos\varphi \left(\frac{\lambda^1+i\lambda^2}{2}\right)_{km} + \sin\varphi \left(\frac{\lambda^4+i\lambda^5}{2}\right)_{km}\right] (\Lambda^{(i)})_{ln} l_{mn} \tag{2·3}$$

is the weak leptonic current. From the lepton-urbaryon correspondence,[4] it will be natural to take the leptonic matrix l_{ij} analogous to t_{ij} in a charge assignment. If we consider the original SUB model[2],**) for the urbaryon fields t_{ij} (see Table II in Paper I), we define

*) If we take the original SUB model, $\Lambda^i = \lambda^i$, $i=0, 1, \cdots, 8$.

**) Our final results are independent of a specific choice of any three-triplet models given in Tables I and II in Paper I.

$$[l_{ij}]\equiv\begin{bmatrix} \nu_e & \nu_\mu & L_1^- \\ e^- & \mu^- & L_2^{--} \\ E^- & M^- & L_3^{--} \end{bmatrix}, \tag{2·4}$$

where E^-, M^-, L_1^-, L_2^{--} and L_3^{--} are the hypothetical leptons (probably heavy) which have not yet been detected. (We will discuss their observability in § 4.) We have assumed the weak interaction Hamiltonian as

$$\mathcal{H}_W=\frac{F_0}{2\sqrt{2}}\sum_{i=0}^{8}\mathcal{J}_\rho^{(i)*}\cdot\mathcal{J}_\rho^{(i)}. \tag{2·5}^{*)}$$

Noticing the relation

$$\sum_{i=1}^{8}(\lambda^i)_{ab}(\lambda^i)_{cd}-\tfrac{4}{3}\delta_{ab}\delta_{cd}=-\sum_{i=1}^{8}(\lambda^i)_{ad}(\lambda^i)_{cb}+\tfrac{4}{3}\delta_{ad}\delta_{cb}$$

and employing the property of the $V-A$ interaction under the Fierz transformation, we can prove that the $|\Delta Y|=1$ nonleptonic interaction in (2·5) leads to the $|\Delta I|=1/2$ rule.

From Eqs. (2·1), ···, (2·5),

$$\mathcal{H}_W(\text{semileptonic};\ \Delta C=0)=\frac{F_0}{\sqrt{2}}g$$

$$\begin{aligned}
&\times\Big[\{(\bar{e}O_\rho\nu_e)\cos\varphi+(\bar{E}O_\rho\nu_e)\sin\varphi\}\Big\{\Big(-2\sqrt{\frac{2}{3}}J_\rho^{1+i2,0}+J_\rho^{1+i2,3}+\frac{1}{\sqrt{3}}J_\rho^{1+i2,8}\Big)\cos\theta_C\\
&\qquad+(1+i2\to4+i5)\sin\theta_C\Big\}\\
&+\{(\bar{\mu}O_\rho\nu_\mu)\cos\varphi+(\bar{M}O_\rho\nu_\mu)\sin\varphi\}\Big\{\Big(-2\sqrt{\frac{2}{3}}J_\rho^{1+i2,0}-J_\rho^{1+i2,3}+\frac{1}{\sqrt{3}}J_\rho^{1+i2,8}\Big)\cos\theta_C\\
&\qquad+(1+i2\to4+i5)\sin\theta_C\Big\}\\
&+\{(\bar{L}_2O_\rho L_1)\cos\varphi+(\bar{L}_3O_\rho L_1)\sin\varphi\}\Big\{\Big(-2\sqrt{\frac{2}{3}}J_\rho^{1+i2,0}-\frac{2}{\sqrt{3}}J_\rho^{1+i2,8}\Big)\cos\theta_C\\
&\qquad+(1+i2\to4+i5)\sin\theta_C\Big\}\\
&+\{(\bar{\mu}O_\rho\nu_e)\cos\varphi+(\bar{M}O_\rho\nu_e)\sin\varphi\}\{J_\rho^{1+i2,1+i2}\cos\theta_C+J_\rho^{4+i5,1+i2}\sin\theta_C\}\\
&+\{(\bar{e}O_\rho\nu_\mu)\cos\varphi+(\bar{E}O_\rho\nu_\mu)\sin\varphi\}\{J_\rho^{1+i2,1-i2}\cos\theta_C+J_\rho^{4+i5,1-i2}\sin\theta_C\}\Big]\\
&+\text{h.c.,}
\end{aligned} \tag{2·6}$$

*) The asterisk $*$ denotes the Hermitian conjugate multiplied by $(-1)^n$, where n is a sum of numbers of 4 and 0 in Lorentz- and $SU(3)''$-indices, respectively.

$$\mathcal{H}_W(\text{semileptonic}\,;\,|\Delta C|=1)=\frac{F_0}{\sqrt{2}}g$$

$$\times\Big[\{(\bar{L}_2O_\rho\nu_e)\cos\varphi+(\bar{L}_3O_\rho\nu_e)\sin\varphi\}\,(J_\rho^{1+i2,4+i5}\cos\theta_C+J_\rho^{4+i5,4+i5}\sin\theta_C)$$

$$+\{(\bar{L}_2O_\rho\nu_\mu)\cos\varphi+(\bar{L}_3O_\rho\nu_\mu)\sin\varphi\}\,(J_\rho^{1+i2,6+i7}\cos\theta_C+J_\rho^{4+i5,6+i7}\sin\theta_C)$$

$$+\{(\bar{e}O_\rho L_1)\cos\varphi+(\bar{E}O_\rho L_1)\sin\varphi\}\,(J_\rho^{1+i2,4-i5}\cos\theta_C+J_\rho^{4+i5,4-i5}\sin\theta_C)$$

$$+\{(\bar{\mu}O_\rho L_1)\cos\varphi+(\bar{M}O_\rho L_1)\sin\varphi\}\,(J_\rho^{1+i2,6-i7}\cos\theta_C+J_\rho^{4+i5,6-i7}\sin\theta_C)\Big]$$

$$+\text{h.c.},\qquad (2\cdot7)$$

where the hadronic currents $J_\rho^{i,j}(i,j=0,1,\cdots,8)$ which transform as $(\mathbf{1}+\mathbf{8},\mathbf{1}+\mathbf{8})$ under the $SU(3)'\times SU(3)''$ are introduced. They are defined as

$$J_\rho^{i,j}(x)\equiv\bar{t}(x)i\gamma_\rho\frac{\lambda^i}{2}\cdot\frac{\Lambda^j}{2}t(x)+\bar{t}(x)i\gamma_\rho\gamma_5\frac{\lambda^i}{2}\cdot\frac{\Lambda^j}{2}t(x)$$

$$\equiv V_\rho^{i,j}(x)+V_{\rho,5}^{i,j}(x).\qquad (2\cdot8)^{*)}$$

The weak coupling constant F_0 is determined so that the strength of purely leptonic interaction in (2·5) may be consistent with the experimental evidence of μ-decay, i.e.,

$$|F_0\cos^2\varphi|=G_0;\;\;G_0=1.02\times10^{-5}\cdot M_p^{-2}.\qquad (2\cdot9)^{**)}$$

On the other hand, we must take

$$|g|=\tfrac{3}{2}|\cos\varphi|,\qquad (2\cdot10)$$

because the experimental evidence of neutron β-decay suggests

$$|gF_0\cos\varphi|=\tfrac{3}{2}G_0\,.$$

Note that the vector form factor of $\langle proton\;|J_\rho(i;0)|neutron\rangle$ is equal to one at zero momentum transfer,[9] where $J_\rho(i;0)$'s, $i=e,\,\mu$, are the $\Delta Y=0$ weak hadronic currents which couple to the electron and the muon currents, respectively. Thus, the semileptonic interaction (2·6) is rewritten as

$$\mathcal{H}_W(\text{semileptonic})=\frac{G_0}{\sqrt{2}}\{(\bar{e}O_\rho\nu_e)J_\rho(e\,;x)+(\bar{\mu}O_\rho\nu_\mu)J_\rho(\mu\,;x)$$

$$+(\bar{\mu}O_\rho\nu_e)J_\rho(\nu_e\mu\,;x)+(\bar{e}O_\rho\nu_\mu)J_\rho(\nu_\mu e\,;x)+\cdots\}+\text{h.c.},$$

where

$$J_\rho(e\,;x)=a\Big\{\Big(-2\sqrt{\frac{2}{3}}J_\rho^{1+i2,0}(x)+J_\rho^{1+i2,3}(x)+\frac{1}{\sqrt{3}}J_\rho^{1+i2,8}(x)\Big)\cos\theta_C$$

$$+(1+i2\to4+i5)\sin\theta_C\Big\},\qquad (2\cdot11)$$

*) The former (the latter) superscripts denote those of $SU(3)'(SU(3)'')$.

**) See (6·4) in Paper I.

$$J_\rho(\mu;x)=a\left\{\left(-2\sqrt{\frac{2}{3}}J_\rho^{1+i2,0}(x)-J_\rho^{1+i2,3}(x)+\frac{1}{\sqrt{3}}J_\rho^{1+i2,8}(x)\right)\cos\theta_C\right.$$
$$\left.+(1+i2\to4+i5)\sin\theta_C\right\}, \tag{2·12}$$

$$J_\rho(\nu_e\mu;x)=a\{J_\rho^{1+i2,1+i2}(x)\cos\theta_C+J_\rho^{4+i5,1+i2}(x)\sin\theta_C\}, \tag{2·13}$$

$$J_\rho(\nu_\mu e;x)=a\{J_\rho^{1+i2,1-i2}(x)\cos\theta_C+J_\rho^{4+i5,1-i2}(x)\sin\theta_C\} \tag{2·14}$$

and

$$|a|=3/2\,.$$

§ 3. Structure functions and sum rules

Let us consider the high-energy neutrino (antineutrino) inclusive reactions with a nucleon target. In particular, we investigate the following charm-number conserving reactions in which only the ordinary leptons participate:

$$\binom{\bar\nu_e}{\nu_e}+N\to\binom{\bar e}{e}+X, \tag{3·1}$$

$$\binom{\bar\nu_\mu}{\nu_\mu}+N\to\binom{\bar\mu}{\mu}+X, \tag{3·2}$$

$$\binom{\bar\nu_e}{\nu_e}+N\to\binom{\bar\mu}{\mu}+X, \tag{3·3}$$

$$\binom{\bar\nu_\mu}{\nu_\mu}+N\to\binom{\bar e}{e}+X, \tag{3·4}$$

where X stands for an unobserved final hadronic state ($C=0$). The lepton-number nonconserving processes (3·3) and (3·4) can occur in the framework of the interaction (2·6).

If the lepton masses are neglected, the cross section for inelastic neutrino scattering from an unpolarized nucleon can be written to lowest order of weak interaction as[5),10),12)]

$$\frac{\pi}{k_0k_0'}\cdot\frac{d\sigma(\bar\nu,\nu)}{dk_0'd\Omega}=\frac{d\sigma(\bar\nu,\nu)}{dq^2d\nu}=\frac{k_0'}{k_0}\cdot\frac{G_0^2}{2\pi}\left[2W_1(\bar\nu,\nu;q^2,\nu)\sin^2\frac{\theta}{2}\right.$$
$$\left.+W_2(\bar\nu,\nu;q^2,\nu)\cos^2\frac{\theta}{2}\pm\frac{k_0+k_0'}{M}W_3(\bar\nu,\nu;q^2,\nu)\sin^2\frac{\theta}{2}\right], \tag{3·5}$$

where $k_0(k_0')$ is the lab. energy of the incident (scattered) lepton, θ the lab. scattering angle, P and M the four-momentum and the mass of the target nucleon, respectively. The square of four-momentum and the energy transferred to the nucleon from the leptonic current are written as

$$q^2=4k_0k_0'\sin^2\frac{\theta}{2}\,,$$

$$\nu = k_0 - k_0' = -\frac{q\cdot P}{M}.$$

The definitions of the structure functions W_i are

$$\begin{aligned}
W_{\rho\sigma}(\bar{\nu}_e, \nu_e) \equiv & \left(\delta_{\rho\sigma} - \frac{q_\rho q_\sigma}{q^2}\right) W_1(\bar{\nu}_e, \nu_e; q^2, \nu) \\
& + \frac{1}{M^2}\left(P_\rho - \frac{q\cdot P}{q^2} q_\rho\right)\left(P_\sigma - \frac{q\cdot P}{q^2} q_\sigma\right) W_2(\bar{\nu}_e, \nu_e; q^2, \nu) \\
& + \frac{1}{2M^2}\epsilon_{\rho\sigma\alpha\beta} q_\alpha P_\beta W_3(\bar{\nu}_e, \nu_e; q^2, \nu) \\
= & \frac{P_0}{M}\int \frac{d^4x}{2\pi} e^{-iq\cdot x}\langle P|[J_\rho^*(\bar{e}, e; x), J_\sigma(\bar{e}, e; 0)]|P\rangle \\
= & \frac{|a|^2}{\pi}\frac{P_0}{M}\int d^4x\, e^{-iq\cdot x}\langle P|\{[V_\rho^{1\pm i2, 9+3}(x), V_\sigma^{1\mp i2, 9+3}(0)] \\
& + [V_\rho^{1\pm i2, 9+3}(x), V_{\sigma,5}^{1\mp i2, 9+3}(0)]\}|P\rangle
\end{aligned} \tag{3·6}$$ *)

for the processes (3·1), and

$$W_{\rho\sigma}(\bar{\nu}_\mu, \nu_\mu) \equiv \left\{\begin{pmatrix}\bar{\nu}_e \\ \nu_e\end{pmatrix} \to \begin{pmatrix}\bar{\nu}_\mu \\ \nu_\mu\end{pmatrix}, 9+3 \to 9-3 \text{ in } (3·6)\right\}, \tag{3·7}$$

$$\begin{aligned}
W_{\rho\sigma}(\bar{\nu}_e\bar{\mu}, \nu_e\mu) \equiv & \left(\delta_{\rho\sigma} - \frac{q_\rho q_\sigma}{q^2}\right) W_1(\bar{\nu}_e\bar{\mu}, \nu_e\mu; q^2, \nu) \\
& + \frac{1}{M^2}\left(P_\rho - \frac{q\cdot P}{q^2} q_\rho\right)\left(P_\sigma - \frac{q\cdot P}{q^2} q_\sigma\right) W_2(\bar{\nu}_e\bar{\mu}, \nu_e\mu; q^2, \nu) \\
& - \frac{1}{2M^2}\varepsilon_{\rho\sigma\alpha\beta} q_\alpha P_\beta W_3(\bar{\nu}_e\bar{\mu}, \nu_e\mu; q^2, \nu) \\
= & \frac{|a|^2}{\pi}\frac{P_0}{M}\int d^4x\, e^{-iq\cdot x}\langle P|\{[V_\rho^{1\pm i2, 1\mp i2}(x), V_\sigma^{1\mp i2, 1\pm i2}(0)] \\
& + [V_\rho^{1\pm i2, 1\mp i2}(x), V_{\sigma,5}^{1\mp i2, 1\pm i2}(0)]\}|P\rangle,
\end{aligned} \tag{3·8}$$

$$\begin{aligned}
W_{\rho\sigma}(\bar{\nu}_\mu\bar{e}, \nu_\mu e) \equiv & \left\{\begin{pmatrix}\bar{\nu}_e\bar{\mu} \\ \nu_e\mu\end{pmatrix} \to \begin{pmatrix}\bar{\nu}_\mu\bar{e} \\ \nu_\mu e\end{pmatrix}; \text{ for the index of} \right. \\
& \left. SU(3)''\ 1\pm i2 \leftrightarrow 1\mp i2 \text{ in } (3·8)\right\}
\end{aligned} \tag{3·9}$$

for the processes (3·2), (3·3) and (3·4), respectively. Here, we have set $\cos^2\theta_C \simeq 1$, $\sin^2\theta_C = 0.05 \simeq 0$ and have introduced $\Lambda^9 \equiv -2\sqrt{2/3}\Lambda^0 + \Lambda^8/\sqrt{3}$ for convenience.

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*) In the arguments of W_i's, $\bar{\nu}_e(\nu_e)$ denote the process in which an incident neutrino is $\bar{\nu}_e(\nu_e)$ and a scattered lepton is $\bar{e}(e)$ in (3·6) and $\bar{\nu}_e\bar{\mu}(\nu_e\mu)$ denotes an incident neutrino $\bar{\nu}_e(\nu_e)$ and a scattered lepton $\bar{\mu}(\mu)$ in (3·8), etc.

Since the Bjorken limit corresponds, as is well known, to the region near L-C in the configuration space,[18] we must calculate the L-C commutators of the currents appearing in (2·8) in order to obtain the behavior of structure functions in the Bjorken limit. Assuming the scale invariance on L-C,[8] we obtain

$$[V_\rho^{i,j}(x),\, V_\sigma^{k,l}(0)] \triangleq -\frac{1}{8\pi}\partial_\alpha[\epsilon(x_0)\delta(x^2)]$$
$$\times [s_{\rho\sigma\alpha\beta}\{(-f\cdot\boldsymbol{f}+d\cdot\boldsymbol{d})^{ikm,jln}A_\beta{}^{m,n}(x|0)+i(f\cdot\boldsymbol{d}+d\cdot\boldsymbol{f})^{ikm,jln}S_\beta{}^{m,n}(x|0)\}$$
$$+\epsilon_{\rho\sigma\alpha\beta}\{(-f\cdot\boldsymbol{f}+d\cdot\boldsymbol{d})^{ikm,jln}S_{\beta,5}^{m,n}(x|0)+i(f\cdot\boldsymbol{d}+d\cdot\boldsymbol{f})^{ikm,jln}A_{\beta,5}^{m,n}(x|0)\}], \tag{3·10}$$

$$[V_\rho^{i,j}(x),\, V_{\sigma,5}^{k,l}(0)] \triangleq -\frac{1}{8\pi}\partial_\alpha[\epsilon(x_0)\delta(x^2)]$$
$$\times [s_{\rho\sigma\alpha\beta}\{(-f\cdot\boldsymbol{f}+d\cdot\boldsymbol{d})^{ikm,jln}A_{\beta,5}^{m,n}(x|0)+i(f\cdot\boldsymbol{d}+d\cdot\boldsymbol{f})^{ikm,jln}S_{\beta,5}^{m,n}(x|0)\}$$
$$+\epsilon_{\rho\sigma\alpha\beta}\{(-f\cdot\boldsymbol{f}+d\cdot\boldsymbol{d})^{ikm,jln}S_\beta{}^{m,n}(x|0)+i(f\cdot\boldsymbol{d}+d\cdot\boldsymbol{f})^{ikm,jln}A_\beta{}^{m,n}(x|0)\}], \tag{3·11}$$

$$[V_{\rho,5}^{i,j}(x),\, V_{\sigma,5}^{k,l}(0)] \triangleq [V_\rho^{i,j}(x),\, V_\sigma^{k,l}(0)], \tag{3·12}$$ *)

where $i, j=0, 1, \cdots, 8$, $s_{\rho\sigma\alpha\beta}=\delta_{\rho\alpha}\delta_{\sigma\beta}+\delta_{\rho\beta}\delta_{\sigma\alpha}-\delta_{\rho\sigma}\delta_{\alpha\beta}$, and

$$\binom{S}{A}_\beta^{m,n}(x|0) \equiv V_\beta{}^{m,n}(x|0) \pm V_\beta{}^{m,n}(0|x)$$

and

$$\binom{S}{A}_{\beta,5}^{m,n}(x|0) \equiv V_{\beta,5}^{m,n}(x|0) \pm V_{\beta,5}^{m,n}(x|0)$$

are symmetrized and anti-symmetrized bilocal vector and axialvector current operators. In the three-triplet model,

$$V_\beta{}^{m,n}(x|0) \equiv \bar{t}(x)\, i\gamma_\beta \frac{\lambda^m}{2}\frac{\Lambda^n}{2}t(0),$$

$$V_{\beta,5}^{m,n}(x|0) \equiv \bar{t}(x)\, i\gamma_\beta\gamma_5 \frac{\lambda^m}{2}\frac{\Lambda^n}{2}t(0).$$

Now, substituting (3·10) and (3·11) into (3·6)∼(3·9), we can easily find that the structure functions scale in the Bjorken limit:

$$\lim_A MW_1(i;q^2,\nu)=F_1^{(i)}(x),$$

$$\lim_A \nu W_2(i;q^2,\nu)=F_2^{(i)}(x),$$

$$\lim_A \nu W_3(i;q^2,\nu)=F_3^{(i)}(x),$$

*) $(-f\cdot\boldsymbol{f}+d\cdot\boldsymbol{d})^{ikm,jln} \equiv -f^{ikm}\cdot\boldsymbol{f}^{jln}+d^{ikm}\cdot\boldsymbol{d}^{jln}$, etc. f^{ikm} and d^{ikm} ($\boldsymbol{f}^{jln}$ and $\boldsymbol{d}^{jln}$) are F- and D-type structure constants of $SU(3)'$ ($SU(3)''$), respectively.

where $(i)=(\bar{\nu}_e, \nu_e, \bar{\nu}_\mu, \nu_\mu, (\bar{\nu}_\mu \bar{e}), (\nu_\mu e), (\bar{\nu}_e \bar{\mu}), (\nu_e \mu))$, $x \equiv q^2/2M\nu$ and $\lim_A \equiv$ limit$(q^2 \to\infty, \nu\to\infty, x$: fixed).[7] If we note that the target nucleon is $SU(3)''$-singlet, the scaling functions have the following forms:

$$F_1^{(\bar{\nu},\nu)}(x)=\frac{1}{2x}F_2^{(\bar{\nu},\nu)}(x), \tag{3·13}$$

and for the processes (3·1) and (3·2),

$$F_2^{(\bar{\nu},\nu)}(x)=4x\{3\tilde{A}^{0,0}(x)+2\sqrt{2}\tilde{A}^{8,0}(x)\pm 2\sqrt{6}\tilde{S}^{3,0}(x)\}, \tag{3·14}$$

$$F_3^{(\bar{\nu},\nu)}(x)=-4\{3\tilde{S}^{0,0}(x)+2\sqrt{2}\tilde{S}^{8,0}(x)\pm 2\sqrt{6}\tilde{A}^{3,0}(x)\} \tag{3·15}$$

and for the processes (3·3) and (3·4),

$$F_2^{(\bar{\nu},\nu)}(x)=\tfrac{4}{3}x\{2\tilde{A}^{0,0}(x)+\sqrt{2}\tilde{A}^{8,0}(x)\pm\sqrt{6}\tilde{S}^{3,0}(x)\}, \tag{3·16}$$

$$F_3^{(\bar{\nu},\nu)}(x)=-\tfrac{4}{3}\{2\tilde{S}^{0,0}(x)+\sqrt{2}\tilde{S}^{8,0}(x)\pm\sqrt{6}\tilde{A}^{3,0}(x)\}, \tag{3·17}$$

irrespectively of the kind of neutrinos. Here, $\tilde{S}^{m,n}(x)$ and $\tilde{A}^{m,n}(x)$ are Fourier components of the spin-averaged expectation values of symmetric and anti-symmetric bilocal currents between one-nucleon states:

$$2P_0\langle P|S^{m,n}(x|0)|P\rangle \equiv P_\beta \int d\xi e^{-i(P\cdot x)\xi}\tilde{S}^{m,n}(\xi)+\text{non-leading terms}, \tag{3·18}$$

$$2P_0\langle P|A^{m,n}(x|0)|P\rangle \equiv P_\beta \int d\xi e^{-i(P\cdot x)\xi}\tilde{A}^{m,n}(\xi)+\text{non-leading terms}. \tag{3·19}$$

Noticing the iso-spin dependent terms, we can simply derive the following symmetric relations among the F_i's:

$$F_i^{\nu p}(x)=F_i^{\bar{\nu}n}(x),\quad F_i^{\bar{\nu}p}=F_i^{\nu n}. \qquad (i=1,2,3) \tag{3·20}$$

By employing the standard technique given by Fritzsch-Gell-Mann,[6] several sum rules are derived from (3·14)~(3·17):

$$\int_0^1 \frac{dx}{x}\{F_2^{\bar{\nu}p}(x)-F_2^{\nu p}(x)\}=\int_0^1 \frac{dx}{x}\{F_2^{\nu n}(x)-F_2^{\nu p}(x)\}$$
$$=3(=\underset{\sim}{2}+1), \tag{3·21}$$

$$\int_0^1 dx\{F_3^{\bar{\nu}p}(x)+F_3^{\nu p}(x)\}=\int_0^1 dx\{F_3^{\nu n}(x)+F_3^{\nu p}(x)\}$$
$$=-9(=-\underset{\sim}{6}-3) \tag{3·22}$$

for the lepton-number conserving processes (3·1) and (3·2), and

$$\int_0^1 \frac{dx}{x}\{F_2^{\bar{\nu}p}(x)-F_2^{\nu p}(x)\}=\int_0^1 \frac{dx}{x}\{F_2^{\nu n}(x)-F_2^{\nu p}(x)\}$$

$$=\frac{3}{2}\left(=\underset{\sim}{0}+\frac{3}{2}\right), \tag{3·23}$$

$$\int_0^1 dx\,\{F_3^{\nu p}(x)+F_3^{\nu p}(x)\}=\int_0^1 dx\{F_3^{\nu n}(x)+F_3^{\nu p}(x)\}$$

$$=-\frac{9}{2}\left(=\underset{\sim}{0}-\frac{9}{2}\right) \tag{3·24}$$

for the lepton-number nonconserving processes (3·3) and (3·4). The figures with the wavy line $\sim$ are values predicted from the quark model. The relation (3·13) is just the Callan-Gross relation[14] which holds commonly in all models in which elementary fields have 1/2-spin.[5] On the other hand, sum rules (3·21) and (3·23) correspond to the Adler sum rule,[10] and (3·22) and (3·24) to the Gross-Smith sum rule.[11] However, the numerical values on the r.h.s.'s of them are different from those derived from the quark model.*)

The reason for the above result can be shown as follows. The target nucleon is assumed to be an $SU(3)''$-singlet, so that the terms which survive on the r.h.s.'s of the L-C commutators (3·10), (3·11) and (3·12) for our processes are those multiplied by $(d\cdot\boldsymbol{d})^{ikm,jln}$ and $(f\cdot\boldsymbol{d})^{ikm,jln}$ with $n=0$. These terms arise not only from the commutators of $SU(3)''$-singlet currents (i.e., $j=l=0$) but also from those of $SU(3)''$-octet currents with $j=l=3, 8$ for the processes (3·1) and (3·2). For the processes (3·3) and (3·4) the terms in question arise from the commutators of $SU(3)''$-octet currents with $j=l=1, 2$ (see Eqs. (3·6)$\sim$(3·9)). The ratio of coefficients**) which are multiplied to $\boldsymbol{d}^{000}$, $\boldsymbol{d}^{jl0}(j=l=3, 8)$ and $\boldsymbol{d}^{jl0}(j=l=1, 2)$ is just 1:1/2:3/4.

§ 4. Discussion

We have considered the charm-number conserving highly inelastic neutrino reactions using our weak charged current and weak interaction (2·5). In particular, we have obtained the sum rules corresponding to the Adler and the Gross-Smith sum rules and found them numerically different from those predicted from the quark model. These sum rules at very high energy in which super hadrons may be produced give one of the crucial tests of our model. We can get similar sum rules for the charm-number changing processes which are induced by the leptonic currents involving the hypothetical leptons. For instance,

$$\int_0^1 \frac{dx}{x}\{F_2(\bar{\nu}_e p\to\bar{L}_2X(C=-1);x)-F_2(\nu_e p\to L_2X(C=1);x)\}=\frac{3}{2},$$

$$\int_0^1 dx\{F_3(\bar{\nu}_e p\to\bar{L}_2X(C=-1);x)+F_3(\nu_e p\to L_2X(C=1);x)\}=-\frac{9}{2}.$$

*) Employing the weak interaction $\mathcal{H}_W$ in the original model ((3·6) in Paper I, $\Delta C=0$), we obtain the following results: r.h.s.'s of (3·21), (3·22), (3·23) and (3·24)$=\underset{\sim}{2}+4$, $-\underset{\sim}{6}-12$, $\underset{\sim}{0}+6$ and $\underset{\sim}{0}-18$, respectively. The charm-number changing interaction has some ambiguity.

**) They come from d^{ikm} and $i\cdot f^{ikm}$ for suitable combinations of i, k and m in $SU(3)'$-space.

(For processes such as $\bar{\nu}_e(\nu_e)+p\to\bar{E}(E)+X(C=0)$ or $\bar{\nu}_e(\nu_e)+p\to\bar{L}_3(L_3)+X(C=\mp 1)$, we obtain the same types of sum rules multiplied by the unknown factor $\tan^2\varphi$.)

Even if we take the three-triplet models other than the original SUB model as the basis of the hadronic current, we are led to the same conclusions. Our consequences are interpreted physically as follows. The structure functions for the neutrino inclusive reactions are given here by the Fourier transform of the spin-averaged expectation value of the hadronic current commutators between one nucleon states (see Eqs. (3·6)～(3·9)). However, they are originally defined as

$$W_{\rho\sigma}(\bar{\nu},\nu)\equiv(2\pi)^3\frac{P_0}{M}\sum_X\langle P|J_\rho^*(\bar{\nu},\nu;0)|X\rangle\langle X|J_\sigma(\bar{\nu},\nu;0)|P\rangle$$
$$\times\delta^4(q+P-P_X), \qquad (4\cdot1)$$

where $|X\rangle$ stands for the unobserved final hadronic states. From the standpoint that $SU(3)''$-nonsinglet super hadrons exist, the completeness relation are written

$$1=\sum_X|X\rangle\langle X|=\sum_{X_0}|X_0\rangle\langle X_0|+\sum_{X_8}|X_8\rangle\langle X_8|+\cdots, \qquad (4\cdot2)$$

where $|X_0\rangle$ indicates the $SU(3)''$-singlet state and $|X_8\rangle$ the $SU(3)''$-octet state; the dots $\cdots$ denotes contributions from other representations of $SU(3)''$.

For the vector and axialvector currents which belong to $(\mathbf{1}+\mathbf{8},\mathbf{1}+\mathbf{8})$ representation of $SU(3)'\times SU(3)''$, we get $\langle X_0|V_\rho^{m,n}(0)|P\rangle=\langle X_0|V_{\rho,5}^{m,n}(0)|P\rangle=0$ for $n\neq0$, but $\langle X_8|V_\rho^{m,n}(0)|P\rangle$ and $\langle X_8|V_{\rho,5}^{m,n}(0)|P\rangle$ with $n\neq0$ are generally nonzero, so in Eq. (4·1) the structure functions get these contributions fron $SU(3)''$-octet final hadronic states. This is the reason why the sum rules (3·21)～(3·24) have the larger values in r.h.s.'s than the predicted ones of the simple quark model.

Thus, if we treat, for instance, the matrix elements of bilinear products of the weak currents, our model yields effects different from the ordinary theory[9] of weak interaction. This is remarkably dissimilar to the fact that the matrix elements of our J_ρ^{hadron} between the $SU(3)''$-singlet states give the same values as the ordinary theory. It must be noticed that, in the energy region where the $SU(3)''$-nonsinglet states might not be produced, our weak interaction model gives the same results as those of the quark scheme for the processes (3·1) and (3·2), and the processes (3·3) and (3·4) never occur at all, as is easily seen. This consequence is just compatible with Lipkin's indication.[15]

Finally, we must remark how we explain the situation that the five hypothetical leptons have not yet been observed. A key point for this problem lies in the fact that weak processes other than those in which the $\bar{\nu}_e e$-, $\bar{\nu}_\mu\mu$-, $\bar{L}_1L_2$-, $\bar{\nu}_e\mu$- and $\bar{\nu}_\mu e$-leptonic currents and the corresponding hadronic ones $J_\rho(\nu_e e;x)$, etc., participate, involve the factor $\tan\varphi$, otherwise they change the charm-number. We have no plausible theories to determine the magnitude of φ and masses of the superhadron states. However, if we assume *ad hoc* $|\cos\varphi|\gg|\sin\varphi|$ as the Cabibbo angle in the hadronic current and appropriately large values of the super-hadron masses, it is possible to interpret the present state of experiments naturally. We must notice that it is very difficult to observe even such processes as those free from the $\tan\varphi$-factor and the super-hadron states; e.g.,

i) $\bar{L}_1+p\to\bar{L}_2+n$,

ii) $e+\bar{\nu}_e$(or $\mu+\bar{\nu}_\mu$)$\to\bar{L}_1+L_2$.

This is because, for the former process, when the masses of the hypothetical leptons are suitably heavy (need not be so heavy, $\sim m_K$), we cannot obtain the incident $\bar{L}_1$ through spontaneous weak decays of low-lying hadrons and, for the latter process, it is not so easy to get the neutrino beam with sufficiently high energy (incidentally, muon target is unrealistic).*)

Although we have not treated the electromagnetic interaction, the best way to search for the hypothetical leptons, if they really exist, will be given by electromagnetic processes such as the photo-production and the electron-positron colliding beam experiments in which a direct pair creation of the leptons might occur, e.g., $\gamma+p\to E^-+E^++p$ and $e^-+e^+\to E^-+E^+$.[16]**) As pointed out by Sakurai,[17] the existence of a relatively low-mass heavy lepton ($m_K<M_l<780$ MeV) is not ruled out by the e^--e^+ experiment.

From the above considerations, we see that the hypothetical leptons, even though they may be heavier than K-mesons, are not necessarily very large. At present, the most stringent restrictions on properties of these heavy leptons are obtained through investigating electromagnetic virtual effects (such as the anomalous magnetic moment of the muon) due to the existence of these leptons.[18]

A more quantitative discussion of the existence of the new leptons is not necessarily appropriate for the main purpose of the present paper, so we may content ourselves with the above qualitative arguments about observability of these leptons.

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*) $\sigma(\bar{\nu}_l+l\to\bar{L}_1+L_2)\sim(G_0^2/\pi)s$, $l=e$, μ. It is very small compared with $\sigma(\nu_l+N)$ at corresponding energy.

**) Once E (the energy of either e^- or e^+ beam) is somewhat larger than the lepton mass M_l, one obtains

$$\sigma(e^-e^+\to l^-l^+)\approx\sigma(e^-e^+\to\mu^-\mu^+)$$
$$\approx 2\times10^{-32}/E^2\,\mathrm{cm}^2;\ E\text{ in GeV-unit.}$$

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