Breakdown of Scaling and Violation of Microcausality

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The observed scaling behaviour for deep inelastic electron scattering suggests that hadrons may be composed of point-like spin one-half constituents and the microcausality may be valid to the deep inside of hadrons.

There arise questions: To what extent does the observed approximate scaling behaviour indicate the validity of the microcausality? What will occur if the microcausality is violated in the deep inside of hadrons? It is the purpose of the present note to discuss these questions.

First we summarize our result in the form of a prediction. *If the microcausality is violated, there will appear a non-scaling behaviour dependent on $(aN_0)^2\nu^2$ as well as on a^2q^2 for deep inelastic electron (muon) scattering. Conversely, if the breakdown of the scaling depends on $(aN_0)^2\nu^2$ as well as on a^2q^2, the violation of the microcausality is one of the possible candidates for its origin.* The parameter a is the fundamental length which represents the region where the microcausality is violated. This result should be compared with that of other scaling breaking models where the beakdown of the scaling depends only on a^2q^2.[1]

In order to define the violation of the microcausality preserving the macrocausality, one must define notions of the neighbourhood of two points in the Minkowski space and that to the lightcone in a relativistic invariant form. One must, for this purpose, introduce a time-like unit vector N_μ.[2] Then, using quantities $R_x = \sqrt{2(xN)^2 - x^2}$ and $L_x = (1/\sqrt{2})\sqrt{R_x^2 - (xN)^2} \pm Nx$, one can define these notions. The origin of N_μ will not be discussed anymore in the present note.

The microcausality can be formulated in the present quantum field theory as the local commutability of the physical quantities. For the electromagnetic interaction of hadrons; for $(x-y)^2 < 0$,

$$[J_\mu(x), J_\nu(y)] = 0. \tag{1}$$

Here, $J_\mu(x)$ is the hadronic electromagnetic current and $x^2 = x_0^2 - \boldsymbol{x}^2$.

Let us define our model for the violation of the microcausality by the following equation:[3]

$$W_{\mu\nu}^a(x, N) = \int d^4\xi\, W_{\mu\nu}(x-\xi)\, \rho(R_{\xi/a}). \tag{2}$$

$W_{\mu\nu}(x)$ is the matrix element of the commutator of the local commutable currents: $W_{\mu\nu}(x) = \langle\beta| [J_\mu(x), J_\nu(0)] |\alpha\rangle$. The parameter a is the fundamental length which represents the region where the microcausality is violated. When $\rho(R_{\xi/a}) \to 0$ rapidly as $R_{\xi/a} \to \infty$, the macrocausality is preserved while the microcausality is violated along the lightcone. This model was applied to the π-N forward dispersion relations in Ref. 3) and compared with the experiment.[4] For later use, some definite forms of the

form factor $\rho(R_{\xi/a})$ are given with their Fourier transforms $\tilde{\rho}(aR_q)$:

$$\rho_1(R_{\xi/a}) = (1/4a^4)\exp[-R_\xi^2/2a^2], \quad (3)$$

$$\hat{\rho}_1(aR_q) = \exp[-(a^2/2)R_q^2], \quad (3')$$

$$\rho_2(R_{\xi/a}) = (1/8\pi^2 a^2 R_\xi^2)\exp[-R_{\xi/a}], \quad (4)$$

$$\tilde{\rho}_2(aR_q) = 1/(1+a^2R_q^2). \quad (4')$$

Now, let us apply the model formulated above to deep inelastic electron (muon) scattering. The cross section of the inclusive lepton scattering is given by a Fourier transform of $W_{\mu\nu}^a(x, N)$ with $\langle\alpha| = \langle\beta| =$ one nucleon state,

$$W_{\mu\nu}^a(q, N) = \int d^4x\, e^{iqx} W_{\mu\nu}^a(x, N)$$
$$= W_{\mu\nu}(q^2, pq)\hat{\rho}(aR_q). \quad (5)$$

So, structure functions $W_i^a(i=1,2)$ are given by the equation

$$W_i^a(q, N) = W_i(q^2, pq)\tilde{\rho}(aR_q). \quad (6)$$

We assume that the usual structure functions $W_i(q^2, pq)$ scale to $F_i(x)$. Then, Eq. (6) becomes in the laboratory frame

$$\left(\frac{M}{\nu}\right)W_i^a(q, N) = F_i(x)\hat{\rho}($$

$$a\sqrt{2\nu^2(N_0 - \sqrt{N_0^2-1}\sqrt{1-q^2/\nu^2}\cos\phi)^2 - q^2}),$$
$$(7)$$

where x is the usual scaling variable, $N_0 = \sqrt{1+N^2}\geq 1$ and ϕ is the angle between space vectors N and q. We have no information about the angle ϕ, but, for the rough estimation of the fundamental length a, the present experimental conditions suggest that we may neglect the $\cos\phi$ term. There appears an $aN_0\nu$-dependent non-scaling behaviour for the structure functions.

Let us estimate the upper bound on the parameter (the product of the fundamental length a and time-component of the time-like unit vector N_μ) in Eq. (3) from the observed approximate scaling behaviour

($\approx 15\%$ approximation in the region $1.5 < q^2 < 10$ GeV^2 and $2 < \nu < 20$ GeV). From the inequality

$$\left|\frac{\nu W_2^a(q, N) - F_2(x)}{F_2(x)}\right|_{\nu\leq 20\,\text{GeV}} < 0.15, \quad (8)$$

we conclude that $a \leq aN_0 < 5\times 10^{-16}$ cm ($1/a \geq 1/aN_0 > 40$ GeV).

Finally, let us estimate the lower bound on the parameter a in Eq. (3) from the second-order weak process $K_L^0 \to \mu^+\mu^-$.[5] We conclude, from the partial decay rate of $K_L^0 \to \mu^+\mu^-$, that $a > 2\times 10^{-15}$ cm ($1/a < 10$ GeV) and $a > 10^{-15}$ cm ($1/a < 20$ GeV) for the current-current case and the IVB case, respectively. These values are larger than that estimated from the approximate scaling behaviour. Thus it may be pointed out that the weak interaction based on the triplet quark model must be excluded[*] if the violation of the microcausality discussed in this note truly exists.

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*) In the quartet model, for example, the most divergent term is not quadratic for the $K_L^0 \to \mu^+\mu^-$ decay.